STUDYING THE FOAMING OF PROTEIN SOLUTIONS BY STOCHASTIC METHODS

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Abstract: A stochastic model studying the formation and destruction of a dispersed protein gas–liquid system (foam) is proposed. The regularities governing the formation of dispersed systems strongly depend on the conditions of a chemical engineering or engineering process, and both the formation of a foam and the destruction of the obtained foam layer occur simultaneously in the process of foam generation. Since a necessary condition for the construction of a stochastic model is the availability of statistical data, which provide the estimation of the number of both forming and bursting bubbles, the method of such a calculation is of topical interest. The model enables the description of the process state at every time moment of the first cycle. One of the characteristics of a foam is its dispersion, so the random variable characterizing the number of bubble per unit volume is introduced to study the processes of foam formation. The mathematical expectation, dispersion, and also the foam destruction rate function are proposed as a basis for the calculation of foaming efficiency characteristics. Since the model is formalized by a set of differential equations, it can also be used in the simulation modeling of the foaming process. The first cycle of the formation and destruction of a protein foam has systematically been studied. The constructed stochastic model has allowed the mathematical expectation and dispersion of the number of protein foam bubbles per unit volume to be calculated at any time moment of gas saturation within the first cycle. It has been shown that the applied numerical solutions of the differential equations are in good agreement with the analytical solutions given by simple formulas convenient for engineering calculations. A method of estimating the model parameters has been developed. The proposed model has allowed the quantitative description of the foaming process both on average and by states. It has been established that the time of the formation of a protein foam in a rotor-stator device at specified process parameters is advisable to be limited by the moment, at which the highest foam destruction rate is attained.

Key words: dispersed protein based gas–liquid systems, stability, stochastic model, probability, random variable moments, differential equations, numerical and analytical solution

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INTRODUCTION

Dispersed gas–liquid systems (foams) in both the liquid and solid form find wide application in different industries (oil-and-gas, food, and metallurgical industries, firefighting, etc.). The mechanism of the foam formation process is complicated due to the combined effect of numerous physicochemical, physicotechnical, and other factors. The regularities governing the formation of dispersed systems strongly depend on the conditions of a chemical engineering or engineering process, and both the formation and destruction of an obtained gas–liquid layer occur simultaneously in the process of foam generation [1–14]. As a consequence, these features complicate to a great extent the mathematical description of the foaming process [3–5, 15–18].

Among the principal characteristics of a foam are the expansion factor (foam-to-solution volumetric ratio), the dispersion (air bubble size), and the stability (time period from the formation of a foam to its partial or complete destruction) [3–8]. The foam stability applicable to any foam independently of its purpose may be considered as a basic characteristic.

It is known that foams based on protein solutions, an increase in the concentration of which improves the foaming properties of a system as a whole, are highly stable [9, 10, 19–24]. The formation of bubbles generally depends on the composition of a foamed solution (foaming agent) and the intensity of a mechanical action, whereas their destruction proceeds under the action of both internal and external forces. For this reason, the entire foam generation process representing a process flow may be considered as a dynamic system of flows or a queueing system. This queueing system will be studied by the methods of stochastic processes and queueing theory [25–27].

The objective of this study is to create a stochastic model, which would systematically describe the processes of foaming in protein solutions and determine the time of the formation of a foam of specified quality.

OBJECTS AND METHODS OF STUDY

Foam formation regularities were studied via the gas saturation of a protein solution (skim milk protein concentrate; protein mass fraction, 4.4%) in a rotor-stator device (GID-100/1 hydrodynamic disperser, which was developed, manufactured, and mounted in the All-Russian Research Institute of Dairy Industry) at a rotor revolution speed from 1750 to 3000 rpm, a working chamber filling coefficient of 0.3, a rotor-stator gap of 0.1 mm, and a processed solution temperature of 13 ± 2°C. Control measurements were performed each three minutes after the freezing of samples in a nitrogen atmosphere and their transmitted-light microscopy on an AxioVert.A1 microscope with an AxioCamERc5 camera and a photo recording block. The number of bubbles was calculated from digital images with the use of corresponding software (the comparison of automatic and manual calculation results for the number of bubbles in the frozen samples shows that the former were underestimated by 18% on average).

Mathematical models were constructed using the tools of probability and stochastic process theory and queueing theory (in combination with the methods of mathematical statistics, mathematical analysis, and differential equations) [25–28].

The effect of the time of the gas saturation of a protein solution was studied using the stochastic model describing the efficiency of the process on average [29–34]. The mathematical expectation (average value) $M_i(t)$ of the random value characterizing the number of particles (bubbles) in the dispersed phase of a foam were considered at a time moment t on condition that their number at the initial time moment was $M_i(0) = i$, and the dispersion of this number was $D_i(t)$, $D_i(0) = 0$, $t \in [0, \infty)$, $i = 0, 1, 2, \ldots$.

A foam generator was considered as an inexhaustible bubble (hereinafter, arrival) generation source characterized by the parameter α, and the bubble destruction (hereinafter, arrival service) was characterized by the parameter β.

Model. A queueing system, to which arrivals entered, was considered. The number of arrivals, which has entered the system for the time t, is a random value ξ satisfying the Poisson process

$$P(\xi = k) = \frac{(\alpha t)^k}{k!} e^{-\alpha t},$$

where α is the intensity determined as the average number of generated bubbles per unit time, $t \in [0, \infty)$, $k = 0, 1, 2, \ldots$.

An arrival that has just entered the system is immediately served. The service time is a random value η distributed in compliance with the exponential law

$$P(\eta < t) = 1 - e^{-\beta t},$$

where $\beta = 1/t_{av}$ is the intensity of service, and t_{av} is the average arrival service time determined as an average bubble life time.

The served arrival leaves the system. It is required to calculate the mathematical expectation $M_i(t)$ and the dispersion $D_i(t)$.

The model was formalized by a "birth-and-death" linear differential equation set, from which a differential equation set for the direct calculation of $M_i(t)$ and $D_i(t)$ [29–34] can be deducted in the form

$$\begin{cases} \frac{d}{dt} M_i(t) + \beta M_i(t) = \alpha \\ \frac{d}{dt} Q_i(t) + 2\beta Q_i(t) = 2\alpha M_i(t) \\ Q_i(t) = D_i(t) + M_i^2(t) - M_i(t), \end{cases} \quad (1)$$

with the initial conditions

$$M_i(0) = i, \; D_i(0) = 0. \quad (2)$$

The solution of set (1) with consideration for Eq. (2) has the form [34]

$$\begin{cases} M_i(t) = \frac{\alpha}{\beta}(1 - e^{-\beta t}) + ie^{-\beta t} \\ D_i(t) = \frac{\alpha}{\beta}(1 - e^{-\beta t}) + ie^{-\beta t}(1 - e^{-\beta t}). \end{cases} \quad (3)$$

If a steady-state regime is attained rather quickly, it is convenient to perform express analysis with the formulas

$$M = \lim_{t \to \infty} M_i(t) = \alpha / \beta,$$

$$D = \lim_{t \to \infty} D_i(t) = \alpha / \beta. \quad (4)$$

Further studies have shown that the formation of bubbles may be considered as a Poisson process [34], and their destruction is a Poisson process only in the first approximation.

Really, it is known that the foam destruction dynamics can conditionally be divided into the three stages: the initial stage is slight destruction, when destruction factors have a minimal effect on a foam, and the destruction rate gradually increases, the active stage is characterized by a considerable increase in the destruction rate up to its maximum value and the greatest effect of each destruction factors, including the effect of their combination, and the attenuation stage is characterized by a decrease in the destruction rate of the stage [3–5].

The stability of a foam depends on the destruction rate, so the parameter $\beta = y(t)$, which has a rate dimension and depends on the foaming time moment, may be considered as its characteristic in our model. As is has been turned out, the destruction of a protein foam is most efficiently described by the function [34]

$$y(t) = B + \frac{A_1}{b_1} \exp(-(t - a)^2 / b_1^2), \quad (5)$$

where A_1, B, a, and b_1 are non-negative numerical parameters found from statistical data, $B > 0$, $y(t) > 0$, $\forall t \in [0, \infty)$.

Moreover, it has been established that the boundaries between the foam destruction stages can be determined by the critical points found for the function $y(t)$ using the standard differential calculation methods. For the convenient application of the function $y(t)$ in engineering calculations, it has been noted that it is similar to the normal distribution density tabulated in the normalized form, so it is written below for convenience of calculations (instead of $y(t)$ at $A_1 = A/\sqrt{\pi},\ b_1 = \sqrt{2}b$) that

$$\beta(t) = B + A\left(\frac{1}{\sqrt{2\pi}b}\exp(-(t-a)^2/(2b^2))\right).$$

RESULTS AND DISCUSSION

The principal technical parameters of most foaming devices are the working body revolution speed, the working chamber filling coefficient, and the processing time and temperature. It is known that an increase in the revolution speed v of the working body of a foam generator can intensify the foaming process (decrease the time of the action on a solution) [4, 5, 9, 10, 12–14, 16, 21], although an intense mechanical action on the formed system also causes its destruction in this case. All these factors in combination have allowed the selection of just this characteristic to study its effect on the dynamics of the formation and destruction of a foam. The values of v = 1750, 2000, 2500, 3000 rpm were considered, and the model parameters corresponding to them were denoted as α_v, β_v. The results of observations at time moments $t = 3m$ min, m = 1, 2, 3, 4, 5, depending on the revolution speed are plotted in Fig. 1.

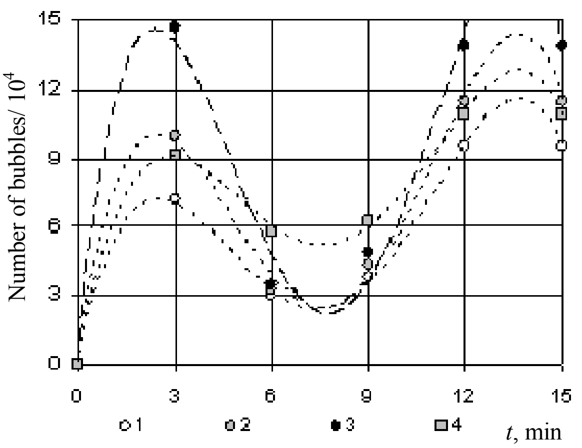

Fig. 1. Number of bubbles per unit volume at time moments $t = 3m$, m = 1, 2, 3, 4, 5 at v of (*1*) 1750, (*2*) 2000, (*3*) 2500, and (*4*) 3000 rpm.

Assuming that there is almost no destruction at the first minute of generation, α_v = 25000, 33250, 49500, 63700 1/min was determined from experimental data. The application of both the average value and the mean-square deviation enabled the estimation of the range of this parameter for each of the foaming processes. The obtained ranges of the parameter α_v are 11500–38400, 12500–53900, 13200–77800, and 22000–105000 1/min, within which the values of this parameter fall during the repeated series of experiments, at v = 1750, 2000, 2500, 3000, respectively.

It is obvious (Fig. 1) that foaming must be limited by 3 min. However, taking into account that just this interval was primarily selected as a time period between measurements in experimental studies, it might occur in reality that an "important moment" had merely been missed, and the repetition of all the series of studies with shorter intervals between measurements with consideration for necessary reproducibility would lead to considerable time, energetic, and material expenditures, it was decided to perform a theoretical study within the framework of already available empirical data and then repeated experimental studies of smaller series.

The formation of a foam was studied from the viewpoint of creating the conditions for its preservation. To accomplish this, it is necessary to minimize the effect of the factors, which lead to the destruction of a foam.

Let us study the foam destruction processes. The parameter (intensity) α in Eqs. (3) is determined as the average number of forming bubbles per unit time, and the parameter β was interpreted also as the average number of bubbles bursting for the same unit time.

The direct estimation of the number of bubbles bursting under the action of different factors presents a certain difficulty, so the values of the parameter β were fitted at fixed time moments. The differences between the generated number of bubbles and their actual number were taken into account. Then the fraction with respect to the actually generated flow of bubbles per unit time was calculated, e.g.,

$$\beta_{2500}(3) = \frac{49500 \cdot 3 - 146324}{146324} : 3 = 0.00496 \text{ 1/min}.$$

The values of the parameter β_v. at time moments $t = 3m$ are plotted in Fig. 2. Their analysis makes it possible to say that the second oscillatory cycle characterized by the growth of the foam destruction rate after the period of descent begins after ≈12 min of the process. For this reason, it is inadvisable to study the foaming process for more than 12 min. Moreover, the highest values of empirical data for all the presented variants correspond to t = 6 min. Taking into consideration the time interval between measurements, it is possible to say that the greatest destructibility of a foam is really attained in the neighborhood of this point on the time axis with a radius of less than 3 min. At this stage, the interval (0, 9) with the center at t = 6 is taken for further studies.

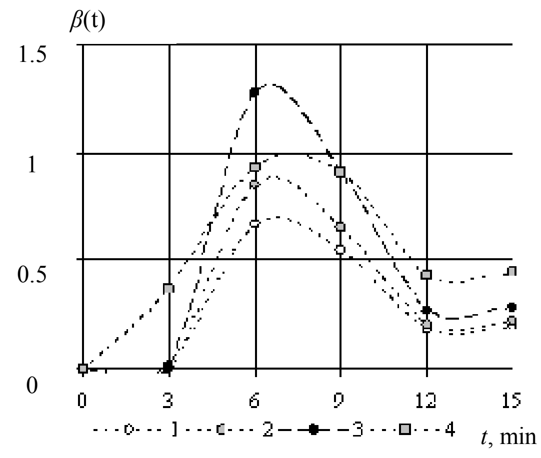

Fig. 2. Parameter β_v versus time $t = 3m$ at v of (*1*) 1750, (*2*) 2000, (*3*) 2500, and (*4*) 3000 rpm.

Since $\beta = 1/t_{av}$ is an averaged characteristic, this raises the question on what time interval should it be considered. Let us cite the values (Fig. 3) calculated for the average number of bubbles per unit foam volume by Eq. (3) for the parameter β calculated on the intervals to 3, 9, and 12 min. It is obvious that the consideration of the averaged constant parameter β enables the description of the number of foam bubbles generated by a rotor-stator device only on a limited time interval. To obtain a more precise description of the considered characteristic $M_i(t)$, it is necessary to express the parameter $\beta = \beta(t)$ in term of time, e.g., by function (5). Thus the specified functional dependence has allowed not only the study of trends in the change of the foam destruction rate, but also the determination of time moments important in this regard and, thereby, the recommended gas saturation time for a protein solution. In other words, the common regularities of the destruction of a protein foam are revealed from the results of studying a particular case.

Hence, taking into account the physical meaning of the parameter $\beta = \beta(t)$, the functional dependence determining it will enables the study of the foam destruction process within the first oscillatory cycle. The values of β_v were approximated by the function $\beta(t)$, the form of which enabled the detection of the time moments important for the foam formation and destruction processes (the physicochemical principles of which are explained in [3–5]) due to the presence of singular points (extrema, inflection points, etc.). Several variants of the approximation of the function $\beta_{2500} = \beta(t)$ in the form of Eq. (5) are shown in Fig. 4, and the error with consideration for the time moment of 12 min variates from 15 to 20%, respectively. When a shorter time interval is considered, the precision increases by nearly two times. Similar results were obtained for the other studied gas saturation processes (ν = 1750, 2000, 3000 rpm).

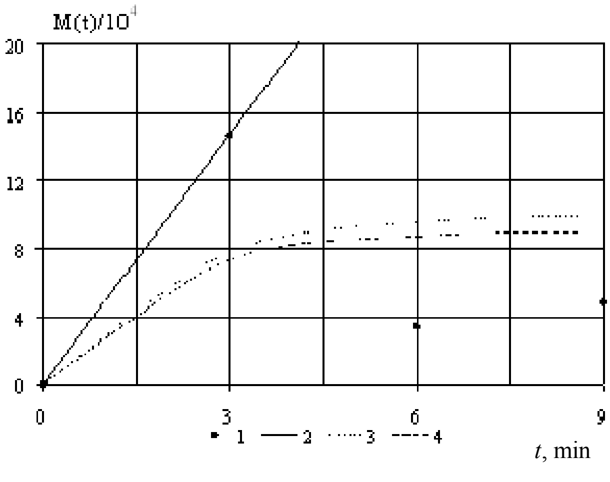

Fig. 3. Number of bubbles versus time t in the rotor-stator processing of a protein solution for (*1*) experimental data and β_{2500} of (*2*) 0.0050, (*3*) 0.4929, and (*4*) 0.5476 1/min.

Let us determine the time moment t0 (located within the interval from 2 to 5 min, as reflected by Fig. 4), from which the foam destruction rate begins to intensively grow.

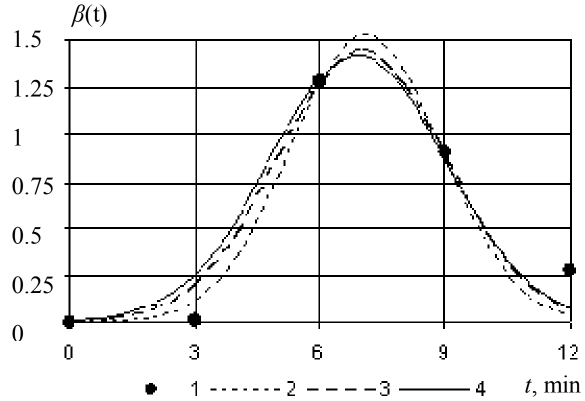

Fig. 4. Time function $\beta(t)$ calculated (*1*) from experimental data and at (*2*) $A = 7.50$, $B = 0.01$, $a = 6.90$, $b = 2.10$, (*3*) $A = 7.20$, $B = 0.01$, $a = 7.00$, $b = 2.00$, and (*4*) $A = 6.90$, $B = 0.01$, $a = 7.10$, $b = 1.80$.

Using the differential calculation methods [34], determining the derivatives $\beta'(t)$, $\beta''(t)$, and the functions $\beta(t)$, and setting them equal to 0, the greatest time t_0, before which the formation of a foam should be stopped (i.e., before the highest foam destruction rate is attained), was found. Hence, the point $t = a$ determines the maximum of the function $\beta(t)$, the inflection point of the function $\beta'(t)$, and the extremum of $\beta''(t)$, $t = a \pm b$ is the inflection points of $\beta(t)$ or the extremum of the function $\beta'(t)$, and $t = a \pm \sqrt{3}b$ is the inflection points of $\beta'(t)$ and the extrema of $\beta''(t)$.

Note that full symmetry is hardly probable in reality (Fig. 4). To estimate the deviation from the symmetry point, if necessary, it is also possible to use the third moment about the mean for the calculation of the asymmetry coefficient. The corresponding formula is derived by solving the differential equation, which is additional to set (1) and obtained by the same method as for the equations of the mathematical expectation and dispersion [29–32]. The found functional dependence $\beta(t)$ was substituted into the set of differential equations describing the mathematical expectation and dispersion to obtain

$$\begin{cases} \frac{d}{dt}M_i(t) + \beta(t)\cdot M_i(t) = \alpha \\ \frac{d}{dt}Q_i(t) + 2\beta(t)\cdot Q_i(t) = 2\alpha M_i(t) \\ Q_i(t) = D_i(t) + M_i^2(t) - M_i(t), \end{cases} \tag{6}$$

with initial condition (2). Note that the derivation of differential equations immediately for the numerical characteristics is insensitive to the form of an equation, i.e., independent of whether the parameters α and β are constants or time functions, although this can not be ensured for the set of differential equations of system probabilities, thus accentuating the advantages of the derivation of differential equations immediately for the numerical characteristics [29–34]. However, the precise analytical solution of set (6) is of no interest due to its cumbersomeness, as it is presented in the form of quadratures or a series, so the specialized Mathematica 5.2 software was used to solve the set by numerical methods. The obtained results are plotted in Fig. 5.

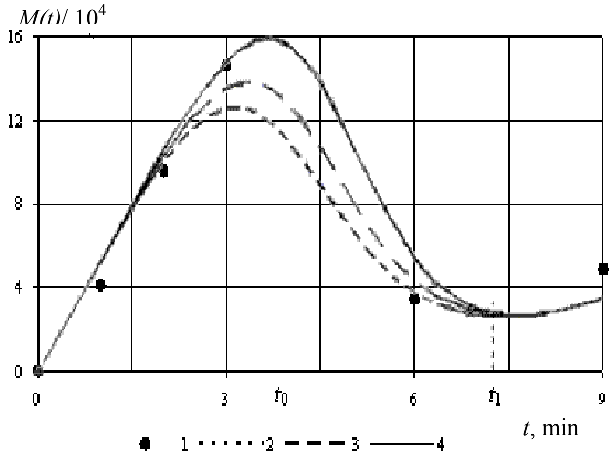

Fig. 5. Number of bubbles versus time t in the rotor-stator processing of a protein solution for (*1*) experimental data, (*2*) $A = 7.50$, $B = 0.01$, $a = 6.90$, $b = 2.10$, (*3*) $A = 7.20$, $B = 0.01$, $a = 7.00$, $b = 2.00$, and (*4*) $A = 6.90$, $B = 0.01$, $a = 7.10$, $b = 1.80$.

From Fig. 4 it can be seen that the empirical curves attain the highest value at a time moment t_0 within the interval [6, 8], i.e., it is unadvisable to perform the process of generation for longer than 8 min, as it appears to be just the time moment, in the neighborhood of which the greatest combined action of all the destruction factors takes place. This is confirmed by Fig. 5. The solution of set (1) with the variable parameter $\beta = \beta(t)$ in the form of Eq. (5) is plotted in Fig. 4. The obtained results for the different values of the parameters of the function $\beta(t)$ describe experimental data at a various precision, which decreases with increasing time interval, and each of them rather precisely describes the overall character of the process on the interval to 9 h. For this reason, after studying the effect of the behavior of the function $\beta(t)$ on the foam destruction dynamics, the foam generation process is considered as a whole and after destruction. From Fig. 5 it can be seen that the interval, on which the foaming process should be interrupted, is [3, t_0]. Similarly, the function $M(t)$ attains the lowest value at a maximum point t_1 of the function $\beta(t)$, and it is obvious that this is inadmissible in the foam generation process.

From the comparison of the results shown in Figs. 4 and 5 it is obvious that the singular points of the function $\beta(t)$ are also singular for the function $M(t)$, so the parameter a is determined by the time moment, at which a foam is maximally destructed, and $t = a \pm b$ is the time moment, at which the destruction rate begins to intensively change, namely, a-b corresponds to an intensive increase in the destruction rate, a+b corresponds to its intensive decrease, $t = a \pm \sqrt{3}b$ are the beginning moments of stable acceleration in the foam destruction rate, i.e., intensive acceleration ($t = a - \sqrt{3}b$) or deceleration ($t = a + \sqrt{3}b$). In this case, the time interval $[a - \sqrt{3}b, a - b]$, to which the terminal time moment of the foaming process must belong, is unequivocally selected. As is evident, the value $t = a - \sqrt{3}b$ is ideal. For the considered function $\beta(t)$ (where $A = 7.50$, $B = 0.01$, $a = 6.90$, $b = 2.10$, $A = 7.20$, $B = 0.01$, $a = 7.00$, $b = 2.00$, and $A = 6.90$, $B = 0.01$, $a = 7.10$, $b = 1.80$, Fig. 4), such time intervals are [3.26, 4.80], [3.54, 5.00], and [3.98, 5.30], and the "ideal" terminal time moments are 3.26, 3.54, and 3.98 min, respectively (note that the error of the description of the protein foam destruction rate by the function $\beta(t)$ on the interval from 0 to 9 min was 15, 18, and 20%, respectively).

Similarly, the intervals [3.45, 4.91], [3.24, 4.76], and [2.36, 4.25] are obtained via the approximation with the function $\beta_v(t)$ at v = 1750, 2000, and 3000 rpm with an error of less than 18% on the interval from 0 to 9 min. The recommended terminal time moments are ≈ 3.5, 3.3, and 2.4 min, respectively.

Note that the difficulties arising in the derivation of the analytical dependence are resolved by finding a numerical solution, which unfortunately does not enables the study of process trends and the system approach to the study of the process, but allows the calculation of numerical results for the average parameters at $\beta = \beta(t)$. In this case, it is almost impossible to obtain any formulas convenient for the application in engineering practice, but, as shown by performed studies, it is quite realistic. The suggested hypothesis on the possibility of the application of Eqs. (3) with constant values of the parameter β averaged over each interval (in our case, these intervals were determined via the regular measurements of samples) was confirmed after the comparison of the experimental data approximation results with the theoretical values, which are the solutions of sets (1) and (6) with initial conditions (2), where $\beta(t) = \overline{\beta}$ and $\beta = \beta(t)$, respectively. In this case, it is evident that the shorter is a considered interval, the smaller is the difference between the averaged parameter β and it real value. Note that the application of differential calculation to the analysis of the function $M_i(t)$ allows (due to that $\beta(t) \neq 0$) the estimation of its maximum value attained at a certain point t_0 from the first equation of set (6) using the inequality

$$M_i(t_0) = \lim_{t \to t_0} \frac{\alpha}{\beta(t)} > 0 \cdot$$

At a constant value of the parameter α, the function $M(t)$ attains the highest value for such an argument, at which the function $\beta(t)$ takes the lowest value on the interval [0, 9). Let us approximate the experimental data on the number of foam bubbles per unit volume by the "pieces" of the function $M(t)$ with the constant parameter $\overline{\beta}$ [25, 28]:

$$\overline{\beta} = \frac{1}{T} \cdot \int_0^T \beta(t) dt \, ,$$

where T is a considered interval. The results plotted in Figs. 4 and 6 demonstrate the nearly identical description of experimental data for both theoretical dependences on the interval [0, 4). The approximation error does not exceed 4%, but an increase in the length of the considered interval appreciably rises it up to 30% and more. Hence, the formulas of set (1) at $\beta(t) = \overline{\beta}$ or simple formulas (3), where β is constant, can be used for the analysis of the foaming process on the interval $[0, t_0)$.

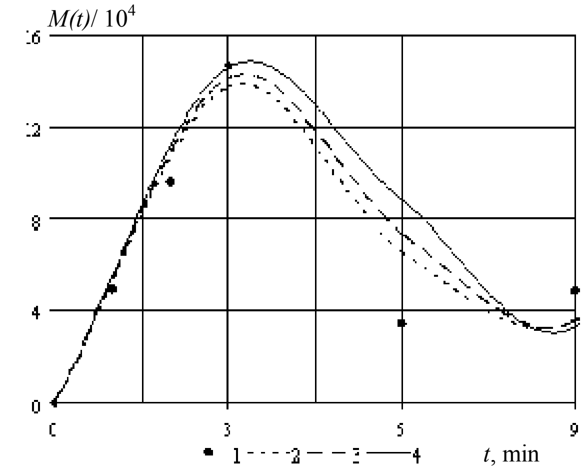

Fig. 6. Number of bubbles versus time t in the rotor-stator processing of a protein solution at $\beta(t) = \overline{\beta}$ for (*1*) experimental data, (*2*) $A = 7.50$, $B = 0.01$, $a = 6.90$, $b = 2.10$, (*3*) $A = 7.20$, $B = 0.01$, $a = 7.00$, $b = 2.00$, and (*4*) $A = 6.90$, $B = 0.01$, $a = 7.10$, $b = 1.80$.

The obtained data were further used as a basis to formulate the constraints of the time of action on a processed mass. Comparing the experimental data and the values of the function $M_i(t)$ expressed by the equation of set (1) with the variable parameter $\beta(t)$ in the form of Eq. (5) on the intervals from 0 to 4.8, 5.0, and 5.3 (Fig. 6), it has been ascertained that the error is from 8 to 2%, respectively, and does not exceed 3% on the interval [0, 4). This allows the error values to be considered as falling within statistical disperancy. The foaming process was studied for t_0 minutes. The interval from 0 to 4 was partitioned with unit steps, at each of which the average value of the parameter β was determined, and the functions $M_i(t)$ and $D_i(t)$ (Fig. 7), which are the solution of set (3), were considered. The preliminary analysis of the experimental data (Fig. 1) shows that the average values of the parameter $\beta(t) = \beta$ on this time interval nearly coincide and equal to 0.010 1/min at a rotor revolution speed of 3000 rpm, except the case corresponding to the generation of a protein concentrate foam. Knowing the capacity of the used equipment for the foaming of a protein solution (the parameter α) and using Eqs. (3) for the description of the process, the range $(M_i(t)-\sqrt{D_i(t)}; M_i(t)+\sqrt{D_i(t)})$ for the number of bubbles per unit volume was obtained (Fig. 7).

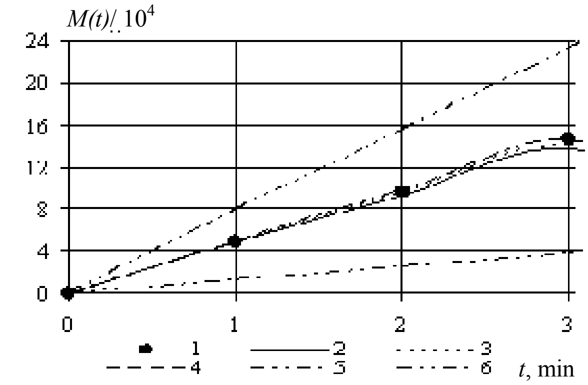

Fig. 7. Number of bubbles versus time t in the rotor-stator processing of a protein solution for (*1*) experimental data, (*2*) $M_0(t)$, $\alpha = 49500$ 1/min, $\beta = \beta(t)$, $A = 7.50$, $B = 0.01$, $a = 6.90$, $b = 2.10$, (3) $M_0(t)$, $\alpha = 49500$ 1/min, $\beta = \beta(t)$, $A = 7.20$, $B = 0.01$, $a = 7.00$, $b = 2.00$, (4) $M_0(t)$, $\alpha = 49500$ 1/min, $\beta = \beta(t)$, $A = 6.90$, $B = 0.01$, $a = 7.10$, $b = 1.80$, and (5,6) boundary $M_0(t)$, $\alpha = 13270$, 77800 1/min, $\beta = 0.010$ 1/min

Note that the experimental data on the number of bubbles per unit volume in the repeated series of experiments on the rotor-stator processing of a protein solution (skim milk protein concentrate with an initial mass fraction of solids of 9.2%) at 1750, 2000, and 2500 rpm (Fig. 7) will also fall into this interval with a near unitary probability. The case corresponding to 3000 rpm is of no interest, as excess hydrodynamic action leads to the considerable destruction of a foam due to the rotation of the device's working body alone.

Hence, the first cycle of the process of protein foam formation and destruction has systematically been studied. The constructed stochastic model has allowed the mathematical expectation and dispersion of the number of protein foam bubbler per unit volume to be determined at a random time moment of gas saturation within the first cycle. It has been shown that the applied numerical methods are in good agreement with the analytical methods, which give simple formulas convenient for engineering calculations. A method of determining the parameters of this model and the dependence of the parameter $\beta(t)$ has been developed. The proposed model has enabled the quantitative description of the foaming process both on average and by states. It has been established that the protein foaming time in a rotor-stator device at specified process parameters is advisable to be limited by the moment, at which the highest foam destruction rate is attained.

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